From Ants to Query Complexity

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Based on joint works with

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Collective transport by Crazy ants

*P. longicornis* ants
Method:
Scent mark detection
A new type of ant trail

1. Individuals lay local trails
2. Group follows them, but not religiously

What is it good for?
To nest
You need to drive a car

You get the map for walking
Navigating with unreliable roadsings

Imagine driving in a foreign country after a hurricane. Most road signs are intact but some have been turned. How can you still get to your destination fast?
The Noisy Advice model

Each node has an advice pointer, which is correct with probability $p$. Otherwise, w.p. $q=1-p$ it points at an arbitrary direction.

How can we reach the nest fast?

Advice per node is fixed!

Very different from advice that can be resampled at any query (allows for busting)

[Feige et al. SICOMP’94]  [Ben-Or and Hassidim FOCS’08]
[Emamjomeh-Zadeh et al. STOC’16]  [Karp and Kleinberg SODA’07]

Very different from worst case placement of wrong advice

[Hanusse et al. PODC’10, TCS’08]
Random Listening (RL)

Random Listening:
• follow advice pointers with a fixed probability $\lambda$, otherwise:
• do a random walk

“Random Walks in Random Environments (RWRE)”
Example: Line graph

Time to pass a trap is exponential in its size
But traps are exponentially rare

Theorem: If $p > 0$ then any listening probability $0 < \lambda < 1 - p$ allows for RL to reach distance $d$ in $O(d)$ time
Predictions

all advice is wrong \[ \Rightarrow \]
time to pass obstacle of size \( x \) is \( \exp(x) \)

most advice is correct \[ \Rightarrow \]
time to pass obstacle of size \( x \) is \( O(x) \)
Predictions

Consistent with step length of 10cm and probability of listening $\lambda = 0.8$

Experimental validations
(performances of living ants)

Consistent with step length of 10cm and probability of listening $\lambda = 0.8$
RL in grids

Situation is “in between” a line and a refreshed advice

Adapting results from RWRE [Snitzman, 2002], we prove:

**Theorem:**
In grids and line graph if $q$ is small enough then there exists a range of listening probabilities to allows for positive speed
Conjecture:
There exists a constant $c$ and a listening probability, s.t. for any graph, RL achieves linear hitting time if mistake $q < c / \Delta$ at every node.
Navigating on Noisy Trees

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To be submitted
Conjecture holds for Trees

1 / \(\Delta\) is a threshold for the noise \(q\) in order for random listening strategies to be efficient

**Theorem**

- There exists a constant \(c\) and a listening probability, s.t. for any tree, RL achieves linear hitting time \(O(d)\) if mistake \(q < c / \Delta\) at every node.

- Consider the complete \(\Delta\)-regular tree. There exists a constant \(c'\), s.t. any RL achieves exponential hitting time (in \(d\)) if mistake \(q > c' / \Delta\).
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Consider the complete $\Delta$-regular tree. There exists a constant $c'$, s.t. any RL achieves exponential hitting time (in $d$) if mistake $q > c' / \Delta$.

What about other algorithms?
Noise regimes

RL

0

\frac{1}{\Delta}

?
$1/\sqrt{\Delta}$ is a lower bound on noise

all algorithms are exponential
Proof sketch

Consider the complete \( \Delta \)-ary tree

Assume:
(1) full advice is given to alg, and
(2) treasure is chosen u.a.r at a leaf

Claim:

**Best algorithm:** counts # of pointers (in the whole tree) pointing at each leaf and checks the leaves in order

Given the claim, time is > the expected number of leaves that “beat” the treasure
Expected number of competitors

Prob that a given leaf is “better looking” than t is small, but there are many leaves!

Prob that \( u \) is “better” than \( t \) is at least \( \frac{q}{\Delta} \cdot q^{d(u,\tau)} \approx q^{2D} \)

There are roughly \( \Delta^D \) leaves whose distance from \( t \) is 2D

Therefore, the expected #leaves that beat \( t \) is at least: \( (q^2\Delta)^D \)
Noise regimes

all algorithms are exponential

RL

0

$\frac{1}{\Delta}$

$\frac{1}{\sqrt{\Delta}}$

1
Noise regimes

Time lower bound of $\Omega(dq\Delta) = \Omega(d\sqrt{\Delta})$
A simple greedy strategy that fails

Walk to the currently most promising node - the one with most pointers pointing to it
A simple greedy strategy that fails

Walk to the currently most promising node - the one with most pointers pointing to it

Consider the complete $\Delta$-ary tree with an extra child to $r$

There are $(\Delta - 1)^D$ leaves $u$. For any of them, prob that $r$ points to $u$, and nobody on the path to $u$ points at $r$ is $>\frac{q}{\Delta} \cdot q^{D-1} \approx q^{D-1}$

So the expected number of nodes visited before the treasure is at least roughly:

$$(q\Delta)^D$$
Theorem: if $q < \frac{1}{\sqrt{\Delta}}$ then there exists a walking algorithm that runs in optimal $O(d\sqrt{\Delta})$ time

Intuition for the construction: Based on a Bayesian approach

Let us make our life easier, and assume:
1. tree structure is known to the algorithm
2. treasure is restricted to leaves
3. $t$ is chosen at random according to a known dist $\theta$

Algorithm: go to the node on the border of what you saw that maximizes the prob that the treasure is a descendant of that node
We want to choose $\Theta$ s.t. the corresponding algorithm will be good against an adversary.

So what would be the good choice of $\Theta$?

The most natural choice is the uniform over all leaves or over all nodes.

This works for complete $\Delta$-ary trees, but fails for general trees :-(

Choosing $\Theta$

We define $\theta$ according to a random walking down process:

Starting at the root, walk down to a child u.a.r. until reaching a leaf.

For a leaf $v$, define $\theta(v)$ as the probability that this process eventually reaches $v$. Our extension of $\theta$ can be interpreted as $\theta(v)$ being the probability that this process passes through $v$. Formally,

$$\theta(\sigma) = 1, \text{ and } \theta(u) = 1/\prod_{w \in [\sigma, u]} \Delta_w$$
Choosing $\theta$

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This works! it gives an algorithm that runs in $O(d\sqrt{\Delta})$ time
The optimal walking algorithm

\[ \theta(\sigma) = 1, \text{ and } \theta(u) = 1/\prod_{w \in [\sigma, u]} \Delta_w \]

\[ \text{score}(u) = \frac{2}{3} \log(\theta(u)) - \sum_{w \in \text{adv}(u)} \log(\Delta_w) \]

Consider all advice discovered so far, and go to a node on the border with highest score.

Note, the algorithm does not need any a priori knowledge of the structure of the tree!
Query Algorithms

What about query algorithms?

all algorithms are exponential (both walking and query)

optimal walking algorithm running in $O(d\sqrt{\Delta})$ time

$0$ $1/\sqrt{\Delta}$ $1/\Delta$ $1$
In the case of refreshable noise, there exists an $O(\log n)$ query algorithm [Feige et al. SICOMP 1994]

It is easy to immolate any protocol on the line by simply querying one of the neighbors of endnotes of the corresponding visited subpath.
Theorem: Lower Bound of $\Omega(\sqrt{\Delta} \cdot \log_{\Delta} n)$ for complete $\Delta$-ary trees

all algorithms are exponential (both walking and query)

optimal walking algorithm running in $O(d \sqrt{\Delta})$ time

lower bound: $\Omega(\sqrt{\Delta} \log_{\Delta} n)$ queries
Query algorithms

Theorem: There is a query algorithm with # of queries $O(\sqrt{\Delta} \cdot \log^2 n)$ on expectation, when $q < 1/\sqrt{\Delta}$

Basic strategy

- Do a separator decomposition.
- For each junction $v$ of the sep tree, apply the walking alg on the subtree $T_v$ of depth $O(\log n)$, until finding a leaf $w$ of $T_v$, for which 80% of the arrows point to it. W.h.p., this will happen by time $O(\sqrt{\Delta} \log n)$.
- Once $w$ is point, the neighbor of $v$ in the sep tree that contains $w$ in its tree is w.h.p the correct separator to continue.
- W.h.p. this finds the treasure within $O(\sqrt{\Delta} \log^2 n)$. If the treasure is not found by this time, do an exhaustive search.

Theorem: There is a query algorithm with # of queries $O(\sqrt{\Delta} \cdot \log n \cdot \loglog n)$ for $\Delta$-regular trees
Summary

A new kind of ant trail

A new kind of model for search in unreliable conditions

RL - A memoryless strategy that is good for grids and trees as long as $q < \frac{c}{\Delta}$

Conjecture: RL is good for any graph as long as $q < \frac{c}{\Delta}$
Summary - on noisy trees

- **Optimal walking algorithm** running in $O(d \sqrt{\Delta})$ time
- **Query algorithm** running in $O(\sqrt{\Delta} \log n \log \log n)$ time for $\Delta$-regular trees
- **Lower bound**: $\Omega(\sqrt{\Delta} \log n)$ queries

All algorithms are exponential (both walking and query).
Open problems

- Solve the random listening conjecture for general graphs
- Find optimal algorithms for other graph families (e.g. expanders?)
Merci!