Knowledge Compilation in Artificial Intelligence and Databases

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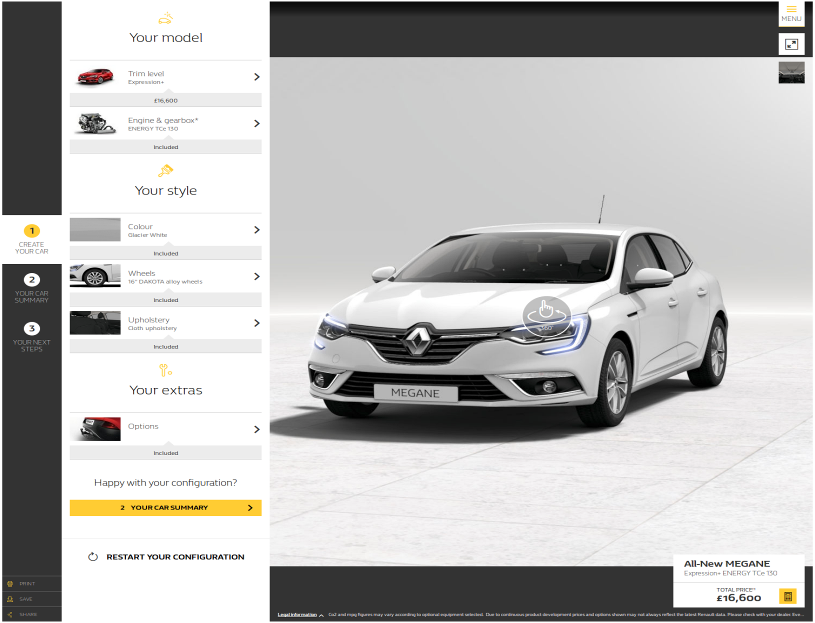
16/03/2017


MENU
Your model
Trim level
Expression+
£16,600
Engine & gearbox*
ENERGY TCe 130
Included
Your style
Colour
Glacier White
Included
Wheels
16" DAKOTA alloy wheels
Included
Upholstery
Cloth upholstery
Included
Your extras
Options
Included
Happy with your configuration?
2 YOUR CAR SUMMARY
RESTART YOUR CONFIGURATION
1 CREATE YOUR CAR
2 YOUR CAR SUMMARY
3 YOUR NEXT STEPS
PRINT
SAVE
SHARE
360°
MEGANE
All-New MEGANE
Expression+ ENERGY TCe 130
TOTAL PRICE
£16,600
Legal information
Co2 and mpg figures may vary according to optional equipment selected. Due to continuous product development prices and options shown may not always reflect the latest Renault data. Please check with your dealer. Eve...

The Configurator as a Computational Problem

- in the applet
  - configure parts of car ($\sim$ 160 parts, some hidden, 10^{26} combinations)
  - constraints on combinations (mostly marketing, 10^{21} possible cars)
  - after partial choice by user, check for legal car (and minimal price, restrict future choices, take back choices, ...)

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- so is there a SAT/CSP-solver in this flash-applet? NO, because...
  - not enough computational power
  - want guaranteed response time

Knowledge compilation and d-DNNF

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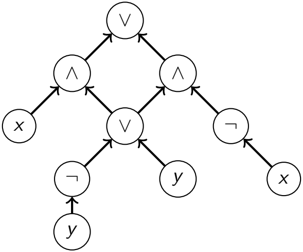

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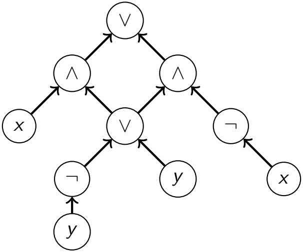

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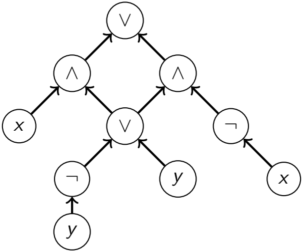

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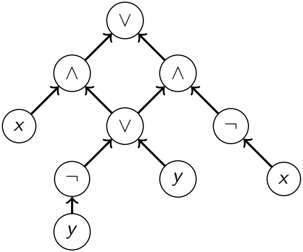

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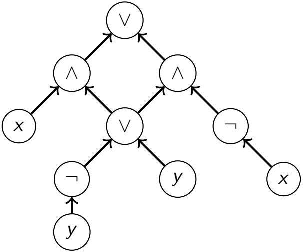

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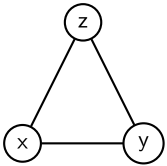
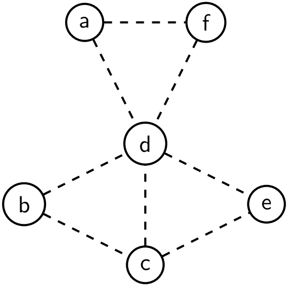
- generalizes other data structures like OBDDs

Probabilistic Embedding





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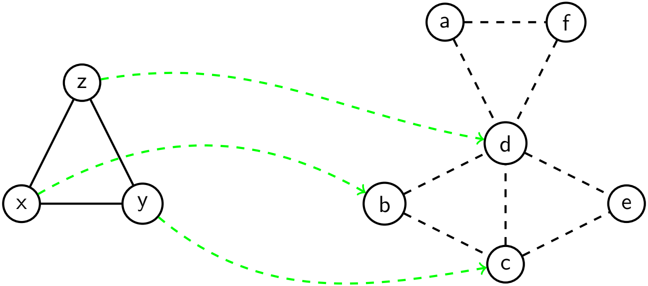

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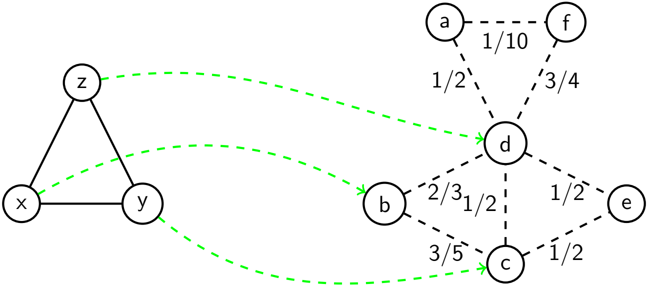

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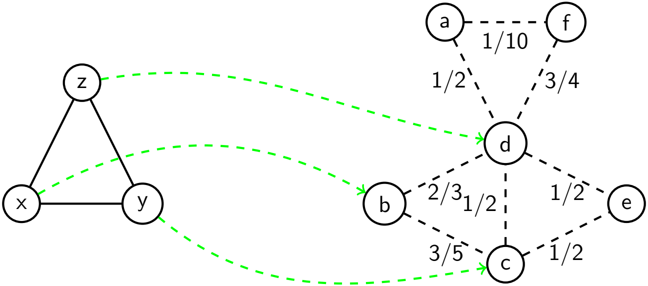
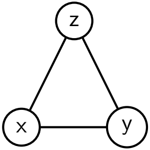
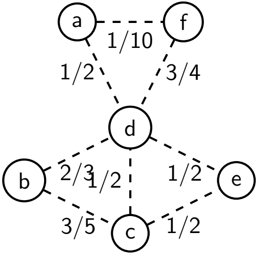


- probability of having mapping that maps edges on edges?
- models problem on probabilistic databases
- #P-hard

The Practical Approach: Grounding





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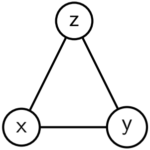
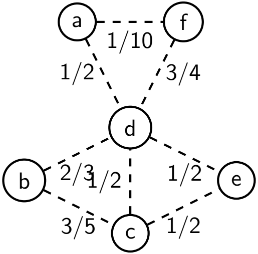
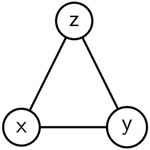
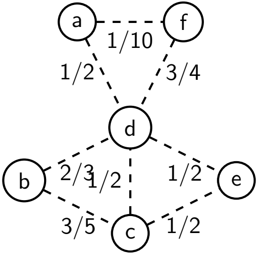




- write down all potential matches, answer “yes” if and only if one of them there

$$\{af, ad, df\}, \{bd, bc, dc\}, \{cd, ce, de\}$$





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$$(x_{af} \wedge x_{ad} \wedge x_{df}) \vee (x_{bd} \wedge x_{bc} \wedge x_{cd}) \vee (x_{cd} \wedge x_{ce} \wedge x_{de})$$

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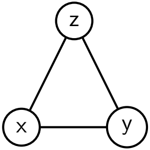

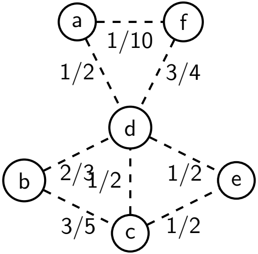

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- compute probability with model counter (typically CNF)

Model Counters: Exhaustive DPLL

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  - backtracking search to find/count solutions of CNF
  - caching (reuse counts of subformulas already solved)
  - components (independent formulas treated independently)
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- so model counters can also compile: sharpSAT $\rightsquigarrow$ Dsharp
- bounds on d-DNNF
  $\approx$ bounds on grounding approach to probabilistic embedding

Lower Bounds for d-DNNF

Theorem (Bova, Capelli, M., Slivovsky, IJCAI 2016)

There are CNF-formulas whose smallest d-DNNF encoding has exponential size.

Communication Complexity

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- Alice and Bob want to evaluate $f(x_1, \ldots, x_n, y_1, \ldots, y_n)$
- Alice knows assignment to $x_1, \ldots, x_n$, Bob knows assignment to $y_1, \ldots, y_n$

Definition: Communication Complexity $CC(f)$

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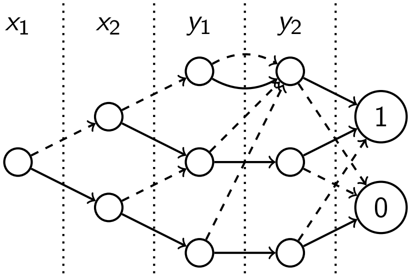
- well studied, many variations, techniques and results

Lower Bounds for OBDD by Communication Complexity



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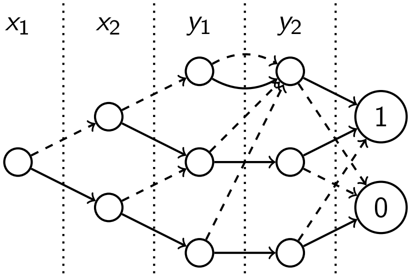

communication protocol from small OBDD:

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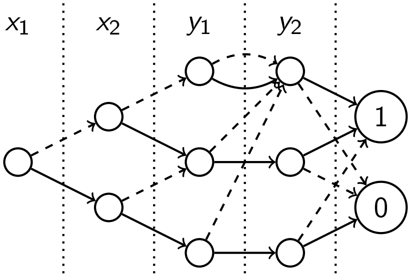

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  $\Rightarrow \log(\#\text{ nodes in middle}) \geq CC(f)$

Generalization to d-DNNF

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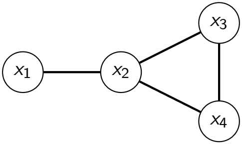
- lets us show lower bounds for d-DNNF "easily" by using CC literature
- several variations of this for other data structures from knowledge compilation

Graphs and CNF

- assign graph to every CNF formula

$$(\neg x_1 \vee x_2) \wedge (x_2 \vee \neg x_3) \wedge (\neg x_2 \vee \neg x_3 \vee x_4)$$



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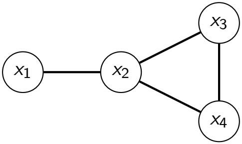

- idea: if graphs are "easy", maybe we can compile efficiently

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- only edge xy: $C_{x,0} = \neg x \wedge y$ and $C_{x,1} = x$
- otherwise x with children $y_1, \ldots, y_s$

$$C_{x,o} = \neg x \wedge C_{y_1,1} \wedge \cdots \wedge C_{y_s,1}$$

$$C_{x,1} = x \wedge (C_{y_1,0} \vee C_{y_1,1}) \wedge \cdots \wedge (C_{y_s,0} \vee C_{y_s,1})$$

compilation result $C = C_{x,0} \vee C_{x_1}$ □

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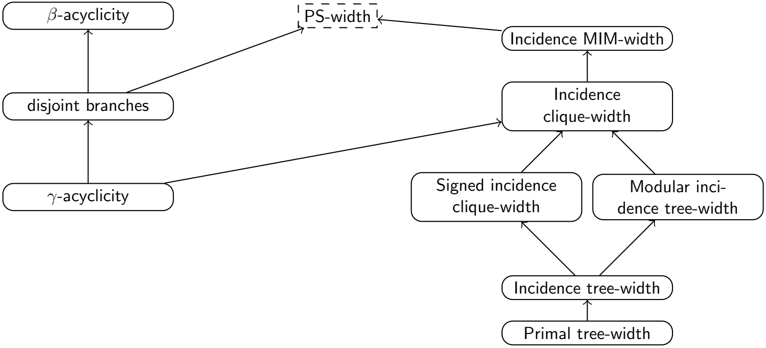

Parameter of the hypergraph **Parameter of the incidence graph**

(Samer, Szeider '10; Paulusma, Slivovsky, Szeider '13; Fisher, Makowsky, Ravve '08; Slivovsky, Szeider '14; Capelli, M, Durand '14; Brault-Baron, Capelli, M '15; Sæther, Telle, Vatshelle '14, Bova, Capelli, M, Slivovsky '15)

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- proved with communication complexity approach
- hard instances random systems of linear equations

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Thank you for your attention!